Moderation and Absorption Times for Negative Pions in Liquid Deuterium*

J. H. DOEDE, R. H. HILDEBRAND, AND M. H. ISRAEL†
Argonne National Laboratory, Argonne, Illinois and The University of Chicago, Chicago, Illinois
(Received 3 August 1964)

The moderation and absorption times for negative pions stopping in liquid deuterium have been measured using the same technique and the same apparatus as were used by Doede, Hildebrand, Israel, and Pyka to obtain the corresponding times in liquid hydrogen. In deuterium the mean time for a negative pion to go from an initial velocity $v_0 \leqslant 0.006c$ to nuclear capture was found to be $(2.1\pm0.5)\times10^{-12}$ sec. The relationship of this capture time to that for liquid hydrogen, $(2.3\pm0.6)\times10^{-12}$ sec, is discussed in detail. Results are also presented for the moderation times of pions with initial velocities between $0.05c$ and $0.006c$.

I. INTRODUCTION

Comparison of the Cascade Process for Pions Stopping in Hydrogen and Deuterium

THE moderation and absorption time for negative pions stopping in liquid hydrogen has been determined experimentally by Fields *et al.*,[1] by Doede *et al.*[2] and by Bierman *et al.*[3] The shortness of this time ($\sim 2\times10^{-12}$ sec) has been explained by Day *et al.*,[4] as due to the high probability of *s*-state capture from states of principal quantum number $n>1$ because of Stark-effect mixing of the angular momentum sublevels of the $\pi - p$ mesic atoms in collisions with hydrogen molecules.

In this paper, we present a measurement of the corresponding time in liquid deuterium. For a discussion of the differences to be expected between deuterium and hydrogen we divide the total moderation and absorption time into intervals using the notation of Doede *et al.*[2]

(1) $\tau_\alpha(v_0)$ = mean time required for a pion to go from a velocity $v_0 > \alpha c$ to a velocity $v = \alpha c (= c/137)$.

(2) $\tau_n(\alpha)$ = mean time spent by a pion in going from a velocity $v = \alpha c$ to the atomic state $n(\approx 4)$ in which nuclear capture predominates.

(3) $\tau_c(n)$ = mean time spent in all states of principal quantum number $n \lesssim 4$ before nuclear capture.

(4) $\tau_c(v_0)$ = total mean time spent by a pion between v_0 and nuclear capture $= \tau_\alpha(v_0) + \tau_n(\alpha) + \tau_c(n)$.

Considering these intervals in order, we find the following differences:

(1) $\tau_\alpha(v_0)$ = *mean time from velocity* $v_0 > \alpha c$ *to velocity* $v = \alpha c$. During the interval $\tau_\alpha(v_0)$ the pion loses energy by the usual processes of ionization and excitation. The rate of energy loss is directly proportional to the frequency of the collisions which the pion undergoes in the stopping medium. Since the atomic density in a deuterium bubble chamber is approximately 10% greater than that in a hydrogen chamber, the time for a pion to reach a velocity αc should be 10% less than the corresponding time in hydrogen. For liquid hydrogen Day[5] has computed that $\tau_\alpha(v_0 = 0.05c) \approx 2.8\times10^{-12}$ sec using the Born approximation. This may be compared with the experimental value $\tau_\alpha(v_0 = 0.05c) = (3.3\pm0.6)10^{-12}$ sec measured by Doede *et al.*[2] Considering the greater density one would expect $\tau_\alpha(v_0 = 0.05c) \cong 3\times10^{-12}$ sec in liquid deuterium.

(2) $\tau_n(\alpha)$ = *mean time from* $v = \alpha c$ *to atomic state* $n \approx 4$. Theoretical estimates[5] show that the time for a pion to go from a velocity $v_0 = \alpha c$ to a bound state of the mesic atom with $n \approx 4$ is about one-half of the total time to go from a velocity $v_0 = \alpha c$ to nuclear capture. During this interval the pion loses energy by several mechanisms:

(a) ionization and elastic Coulomb collision with atoms in the stopping medium,

(b) mesic molecule formation and then molecular dissociation into the mesic atom with $n \approx 30$,

(c) collisions of the mesic atom with molecules of the surrounding medium in which the molecules are dissociated and the mesic atom goes from $n \approx 30$ to $n \approx 15$,

(d) collisional Auger transitions which de-excite the mesic atom from $n \approx 15$ to $n \approx 4$.

In the velocity range from $\sim \alpha c (= c/137)$ to $\sim 10^{-4}c$, the pion loses part of its energy by ionization of molecules and part by elastic Coulomb collisions with nuclei in the liquid. In a nuclear collision, a deuteron, because of its greater mass, absorbs only about half as much energy as a proton, hence in deuterium the rate of energy loss due to this mechanism is only half that in hydrogen. Wightman[6] calculates that the time spent by the pion in this velocity range is 0.74×10^{-12} sec in liquid hydrogen and 1.1×10^{-12} sec in liquid

* This work supported in part by the U. S. Atomic Energy Commission, in part by the National Science Foundation, and in part by the U. S. Office of Naval Research.

† Present address: California Institute of Technology, Pasadena, California.

[1] T. H. Fields, G. B. Yodh, M. Derrick, and J. G. Fetkovich, Phys. Rev. Letters **5**, 69 (1960).

[2] J. H. Doede, R. H. Hildebrand, M. H. Israel, and M. R. Pyka, Phys. Rev. **129**, 2808 (1963).

[3] E. Bierman, S. Taylor, E. L. Koller, P. Stamer, and T. Heutter, Phys. Letters **4**, 351 (1963).

[4] T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. Letters **3**, 61 (1959); and Phys. Rev. **118**, 864 (1960).

[5] T. B. Day, Bull. Am. Phys. Soc. **5**, 225 (1960); and University of Maryland Physics Department Technical Report No. 175, 1960 (unpublished).

[6] A. S. Wightman, Phys. Rev. **77**, 521 (1950).

deuterium for the same atomic density. Allowing for the greater atomic density of liquid deuterium this time becomes approximately 1×10^{-12} sec.

In hydrogen when the pion has slowed to $\sim10^{-4}c$, it is captured into a $(\pi^- H_2)^+$ molecule which dissociates[5] in 10^{-13} to 10^{-14} sec, leaving a π mesic atom with principal quantum number $n\approx30$. This atom de-excites to $n\approx15$, losing energy predominantly in collisions in which H_2 molecules are dissociated. Since the dissociation energy for deuterium (4.553 eV) is only slightly greater than that for hydrogen (4.476 eV), and since the time for the pion to go from $n\approx30$ to $n\approx15$ (in hydrogen) is only $\sim0.2\times10^{-12}$ sec,[5] the difference in this interval for hydrogen and deuterium is a negligible fraction of $\tau_n(\alpha)$. The difference due to the higher atomic density of deuterium may also be neglected in this short interval.

Collisional Auger transitions are the dominant mode of de-excitation of the pion mesic atom as it cascades down from the $n\approx15$ to the $n\approx4$ atomic level. From the analysis of Leon and Bethe[7] we estimate that a pion in hydrogen will take 1.25×10^{-12} sec[8] to go from $n\approx15$ to $n\approx4$.

The time required for Auger de-excitation varies as the square of the reduced mass of the mesic atom and inversely with the atomic density. Comparing hydrogen and deuterium for this interval we find that the effects of the difference in reduced mass and atomic density on $\tau_n(\alpha)$ are opposite in sign and nearly equal in magnitude.

We see that the difference (0.3×10^{-12} sec) expected between the times $\tau_n(\alpha)$ for liquid deuterium and for liquid hydrogen arises mainly from the difference in the time to go from $v\approx\alpha c$ to a bound mesic atom of $n\approx30$.

(3) $\tau_c(n\lesssim4)$ = *mean time from atomic state* $n\approx4$ *to nuclear capture.* Almost all of the pions will undergo nuclear absorption when they reach the s state of principal quantum number $n=4$ or 3. As noted above, the pion has a high probability of reaching the s state of the nth level due to Stark-effect mixing of angular momentum sublevels during collisions in the liquid. A comparison of the rates of nuclear absorption in hydrogen and deuterium can be separated into factors which depend on the atomic state of the pion and those which are independent of the atomic state.

A comparison of the atomic state independent factors for deuterium and hydrogen may be obtained from the cross sections for low-energy pion production reactions following the treatment of Brueckner, Serber, and Watson.[9] The procedure may be outlined as follows:

The ratio R of total absorption rates $\Lambda_{s(\mathrm{D})}$ and $\Lambda_{s(\mathrm{H})}$ for pions in the $1s$ state of deuterium and hydrogen

$$R=\frac{\Lambda_{s(D)}}{\Lambda_{s(\mathrm{H})}}=\frac{\Lambda(\pi^-+d\rightarrow 2n)+\Lambda(\pi^-+d\rightarrow 2n+\gamma)}{\Lambda(\pi^-+p\rightarrow n+\pi^0)+\Lambda(\pi^-+p\rightarrow n+\gamma)} \quad (1)$$

is given by

$$R=T(D+1)/(P+1)\,, \quad (2)$$

where

$$T=\frac{\Lambda(\pi^-+d\rightarrow 2n+\gamma)}{\Lambda(\pi^-+p\rightarrow n+\gamma)} \quad (3)$$

and P and D are the well-known Panofsky ratios

$$P=\frac{\Lambda(\pi^-+p\rightarrow n+\pi^0)}{\Lambda(\pi^-+p\rightarrow n+\gamma)} \quad (4)$$

and

$$D=\frac{\Lambda(\pi^-+d\rightarrow 2n)}{\Lambda(\pi^-+d\rightarrow 2n+\gamma)}\,. \quad (5)$$

Since accurate measurements are available for P[10] and D,[11,12] we need only consider the ratio T. This ratio, which may be written

$$T=\frac{1}{D}\frac{\Lambda(\pi^-+d\rightarrow 2n)}{\Lambda(\pi^-+p\rightarrow n+\gamma)}\,,$$

may be obtained from the low-energy cross sections

$$\sigma(\pi^-+d\rightarrow n+n) \quad (6)$$

and

$$\sigma(\pi^-+p\rightarrow n+\gamma)\,, \quad (7)$$

using the relationship

$$\Lambda=|\Psi(0)|^2 v_{\pi p(d)}\sigma\,,$$

where $\Psi(0)$ is the $1s$ bound-state wave function evaluated at the proton (or deuteron) and σ is the cross section (6) [or (7)] corresponding to a relative velocity $v_{\pi p(d)}$ of the pion and the proton (or deuteron). We assume that the product $v\sigma$ is nearly the same for the bound state as for low positive energies. Since the cross sections for (6) and (7) have not been accurately measured, we use detailed balancing to obtain them from the cross sections for the inverse reactions

$$n+n\rightarrow\pi^-+d \quad (8)$$

and

$$\gamma+n\rightarrow\pi^-+p\,. \quad (9)$$

However, the cross sections for (8) and (9) are not known directly. We may obtain the cross section for

---

[7] M. Leon and H. A. Bethe, Phys. Rev. **127**, 636 (1962).

[8] Leon and Bethe performed their calculations for hydrogen of 4.3×10^{22} atoms/cm^3 density; hydrogen in a bubble chamber has a density of 3.4×10^{22} atoms/cm^3. This has been taken into account in obtaining the value $\sim1.25\times10^{-12}$ sec.

[9] K. Brueckner, R. Serber, and K. Watson, Phys. Rev. **81**, 575 (1951).

[10] V. T. Cocconi, T. Fazzini, G. Fidecaro, M. Legros, N. H. Lipman, and A. W. Merrison, Nuovo Cimento **22**, 494 (1961).

[11] P. K. Kloeppel, EFINS Report No. (6-64) (to be published).

[12] J. Ryan, University of California Lawrence Radiation Laboratory Report No. UCRL 9884, 1962 (unpublished).

(8) from the measured cross section[13] for the reaction

$$p+p \to \pi^+ + d \tag{10}$$

by charge symmetry after Coulomb correction terms are taken into account.

We may obtain the cross section for reaction (9) from the measured cross section $\sigma(\gamma+p\to\pi^+ + n)$ using the relationship

$$\frac{\sigma(\gamma+n\to\pi^- + p)}{\sigma(\gamma+p\to\pi^+ + n)} = r, \tag{11}$$

where r, which is defined by (11), is derived from the ratio of π^- to π^+ in photopion production from deuterium near threshold.[14]

Combining the steps outlined above, we find the expression

$$T=\frac{1}{D}\times\frac{1}{r}\frac{\sigma(p+p\to\pi^+ + d)v_{\pi d}}{\sigma(\gamma+p\to\pi^+ + n)v_{\pi n}} \times\frac{1}{3}\frac{M_p}{m_\pi}\left(1+\frac{m_\pi}{4M_p}\right)\left[\left(1+\frac{m_\pi}{M_p}\right)\Big/\left(1+\frac{m_\pi}{2M_p}\right)\right]. \tag{12}$$

In evaluating (12) we have used the following figures:

$$D=3.02\pm0.10,^{15} \quad r=1.3\pm0.1,^{14,16}$$

$$\sigma(p+p\to\pi^+ + d)=(0.155\pm0.017)\eta \text{ mb},^{13}$$

$$\sigma(\gamma+p\to\pi^+ + n)=(0.197\pm0.014)\eta \text{ mb}.^{17}$$

With these figures we obtain

$$T=0.51\pm0.06 \tag{13}$$

and using[10] $P=1.53\pm0.02$ we get

$$R=0.80\pm0.10. \tag{14}$$

The above value of T may be compared with the value obtained by Traxler[18] in a direct calculation of T as defined in (3) taking into account the effects of the exclusion principle and the final state interactions between the two neutrons. Traxler's value is

$$T=0.83\pm0.08 \tag{15}$$

from which we obtain

$$R=1.32\pm0.13. \tag{16}$$

Let us now consider the factors which depend on the atomic state from which the pion is captured. If the total capture rate from the 1s state of a given mesic atom is Λ_s, then the capture rate in the ns state is $\Lambda_s n^{-3}$. Now if Stark mixing produced a statistical distribution of the population of angular momentum states, the chance of finding a pion in the s state would be n^{-2} so that the capture rate from the nth level would be $\Lambda_s n^{-5}$. However, for pions of $n\lesssim 6$ the rate of nuclear absorption is comparable to the rate of Stark mixing so that the population of the ns states is depleted between collisions. Accordingly, the effective absorption rate[7] is

$$\Lambda_{\text{eff}}(n)=\Lambda_{\text{St}}(n)n^{-2}\{1-\exp[-(\Lambda_s n^{-3})/\Lambda_{\text{St}}(n)]\}, \tag{17}$$

where $\Lambda_{\text{St}}(n)$ is the rate of Stark mixing. Λ_{St} is a direct function of the frequency of collisions of the pion mesic atom. Since the frequency of collisions depends directly on the velocity of the mesic atoms and on the density of the stopping medium, $\Lambda_{\text{St(D)}}(n)$ for deuterium will be approximately $1.1/\sqrt{2}$ times smaller than the rate $\Lambda_{\text{St(H)}}(n)$ for hydrogen at the same value of n. It follows from this and from the form of Eq. (17) that the ratios $[\Lambda_{\text{eff}}(n_i)/\Lambda_{\text{eff}}(n_j)]_{i\neq j}$ will be different for hydrogen and deuterium. For example, assuming a rate $\Lambda_s=1.1\times10^{15}$ sec^{-1} for hydrogen and the theoretical value for R, one obtains $\Lambda_{\text{eff(D)}}(4)\approx0.97\Lambda_{\text{eff(H)}}(4)$ and $\Lambda_{\text{eff(D)}}(3)\approx0.78\Lambda_{\text{eff(H)}}(3)$.[19]

Table I presents additional figures for Λ_{eff} based on the two different values of T discussed above [Eq. (13) and (17)]. For the $n=3$ level of deuterium, the rate of nuclear capture $\Lambda_s n^{-3}$ is much larger than the rate of Stark mixing $\Lambda_{\text{St}}(n)$ so that $\Lambda_{\text{eff(D)}}(n=3)$ is controlled primarily by Λ_{St}. Furthermore, since the capture time $\tau_c(n)$ from $n\lesssim4$ is only about 40% of the total capture time $\tau_c(\alpha)$ from $v_\pi\approx\alpha c$, the measurement of $\tau_c(\alpha,\text{D})/\tau_c(\alpha,H)$ will be insensitive to $\Lambda_s(\text{D})/\Lambda_s(\text{H})$.

Although it has been necessary to consider the atomic state independent capture rate Λ_s in order to estimate

TABLE I. Values of Λ_{eff}. Rates in units of 10^{12} sec^{-1}.

	$T=0.51$[a] $R=0.80$	$T=0.83$[b] $R=1.32$
$\Lambda_{\text{eff(D)}}(n=4)$	0.50	0.62
$\Lambda_{\text{eff(D)}}(n=4)/\Lambda_{\text{eff(H)}}(n=4)$	0.79	0.97
$\Lambda_{\text{eff(D)}}(n=3)$	0.57	0.57
$\Lambda_{\text{eff(D)}}(n=3)/\Lambda_{\text{eff(H)}}(n=3)$	0.78	0.78
Fraction of those reaching $n=4$ in D_2 which are captured.	0.49	0.54
Fraction of those reaching $n=3$ in D_2 which are captured.	0.90	0.90

[a] This value of T is obtained from Eq. (12) using the figures immediately following Eq. (12) in the text.
[b] This value of T is obtained from Traxler's direct calculation (Ref. 18) [see Eq. (15) of text].

[13] F. S. Crawford, Jr., and M. L. Stevenson, Phys. Rev. **97**, 1305 (1955). The figure $(0.138\pm0.015)\eta$ given in this reference has been corrected for Coulomb effects. We are indebted to P. K. Kloeppel for bringing this correction to our attention.
[14] W. R. Hogg, Proc. Phys. Soc. (London) **80**, 729 (1962).
[15] This is the average of the values of D reported in Refs. 11 and 12.
[16] J. Hamilton and W. S. Woolcock, Phys. Rev. **118**, 291 (1960).
[17] G. M. Lewis, R. E. Azerma, E. Gaba Thuler, D. W. G. S. Leith, and W. R. Hogg, Phys. Rev. **125**, 378 (1962).
[18] R. H. Traxler, University of California Radiation Laboratory Report No. UCRL 10417, 1962 (unpublished).
[19] Using values of Λ_{St} taken from the tabulation in Ref. 7.

the capture time $\tau_c(\alpha)$ we see that the latter is neither a simple nor a sensitive measure of the former.

(4) $\tau_c(v_0)\approx\tau_\alpha(v_0)+\tau_n(\alpha)+\tau_c(n)$. The total time for a pion to go from a velocity v_0 to nuclear capture in liquid deuterium is thus modified, relative to the time required in liquid hydrogen, as follows:

$$\tau_{c(\mathrm{D})}(v_0)=0.90\tau_{\alpha(\mathrm{H})}(v_0)+1.1\tau_{n(\mathrm{H})}(\alpha)+R_{\mathrm{eff}}\tau_{c(\mathrm{H})}(n)\,,$$

where

$$R_{\mathrm{eff}}=\tau_{c(\mathrm{D})}(n)/\tau_{c(\mathrm{H})}(n)\,.$$

Values of R_{eff} corresponding to the values of T [Eqs. (13) and (15) above] are presented in the last row of Table II. Previous experiments in liquid hydrogen[2] have reported a value of

$$\tau_{c(\mathrm{H})}(\alpha)=\tau_{n(\mathrm{H})}(\alpha)+\tau_{c(\mathrm{H})}(n)=(2.3\pm0.6)10^{-12}\ \mathrm{sec}.$$

The corresponding time in liquid deuterium should then be given by

$$\tau_{c(\mathrm{D})}(\alpha)=1.1\tau_{n(\mathrm{H})}(\alpha)+R_{\mathrm{eff}}\tau_{c(\mathrm{H})}(n)\,. \tag{18}$$

Theoretical estimates of these times in liquid hydrogen as displayed in Table II indicate that

$$\tau_{n(\mathrm{H})}(\alpha)\cong1.5\tau_{c(\mathrm{H})}(n)\,. \tag{19}$$

Using this result and Eq. (18), predicted rates for liquid deuterium have been calculated for the assumed values of R_{eff}. These are presented in Table II. In view of the many processes which must be considered, it is not surprising that the estimated values for $\tau_c(\alpha)$ in hydrogen differ from the measured value.[2] Estimates of the ratio $\tau_{\mathrm{D}_2}/\tau_{\mathrm{H}_2}$ should be more reliable than estimates of the individual capture times, provided no significant steps have been omitted which depend on the isotope. As a test of the cascade process as outlined above we have measured $\tau_c(\alpha)$ for deuterium.

II. EXPERIMENT

The technique and apparatus for this experiment were the same as those used by Doede *et al.*[2] to measure the time for pion capture in hydrogen. For a pion which decays so that the muon is emitted into the backward hemisphere, the pion velocity is uniquely determined by the muon range and its angle relative to the pion direction. With the distribution of these pion velocities at decay and the total number of captured pions we may calculate the mean time, $\tau_c(v_0)$, for pions to go from a velocity v_0 to nuclear capture using the expression

$$\tau_c(v_0)=[4\pi/\Omega(v_0)](n/N)\tau_\pi\,, \tag{20}$$

where $\Omega(v_0)$ is the solid angle in the pion system corresponding to laboratory angles between 180° and some minimum angle $\theta_{\min}\geqslant90°$ selected for scanning convenience (for this experiment $\theta_{\min}=110°$), n is the number of pions which decay with laboratory $\pi-\mu$ angle $\geqslant\theta_{\min}$ at a velocity $\leqslant v_0$, N is the number of

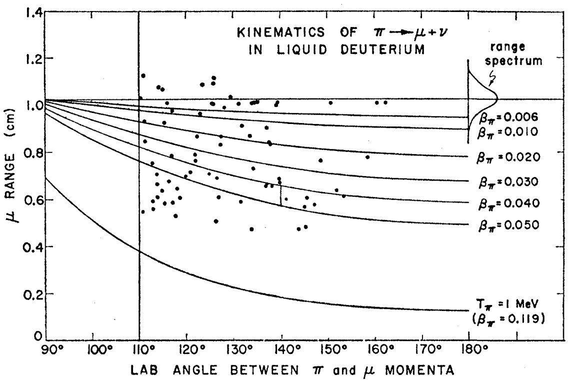

FIG. 1. Kinematics of $\pi^-{-}\mu^-$ decay in flight in liquid deuterium of density 12.63×10^{-2} g/cm³. Each dot represents one decay found in 1 056 000 stops. Also shown is a μ^+ range spectrum determined from measurements of 345 positive muons emitted by positive pions decaying at rest.

pions captured in the chamber = 1 056 500, and τ_π is the mean lifetime of the pion $=2.56\times10^{-8}$ sec.[20]

In this experiment, the Chicago 9-liter bubble chamber filled with liquid deuterium of density 12.64×10^{-2} g/cm³ was exposed to a 68 MeV/c negative pion beam from the University of Chicago cyclotron. The chamber was operated in a magnetic field of 24.7 kG. 1 056 500 pions were stopped in a region whose boundaries were at least 1.5 cm from the chamber walls.

Pion decays were accepted by the scanners if (1) the included angle between the pion and muon tracks was less than 90°, (2) the muon range was greater than 0.5 cm, and (3) the muon had a decay electron. All frames were scanned twice with an over-all scanning efficiency greater than 99%. Measurement of accepted events was done on the University of Chicago "Olivia" digital measuring machine. The data reduction program calculated the angle at the $\pi^-{-}\mu^-$ vertex, the range of the muon, and the position of the vertex in the chamber. The uncertainty in the measurement of the angle at the $\pi-\mu$ vertex was $\leqslant2°$. The ranges of positive muons from 345 positive pion decays were measured to determine the precision of the negative muon range measurement. The kinematics of $\pi^-\to\mu^-$ decay events in the backward hemisphere, with a vertex in the central region of the chamber, are indicated in Fig. 1 along with the experimental positive muon spectrum. Together they indicate that determination of pion velocities down to $v_0\approx0.006c$ is possible. Figure 2 shows the μ^- range spectrum disregarding $\beta_\pi(=v_{\pi/c})$ with the normalized μ^+ spectrum superimposed.

The number of events with $v_0\leqslant0.006c$ must be corrected for events which appear to fall outside this interval, especially near $\theta_{\min}$, because of the spread in range measurements. This correction is made by plot-

[20] J. Ashkin, T. Fazzini, G. Fidecaro, Y. Goldschmidt-Clermont, N. A. Lipman, A. W. Merrison, and H. Paul, Nuovo Cimento **16**, 490 (1960).

TABLE II. Summary of capture processes.

Symbol	Interval	Processes	Factors influencing rate	D_2 versus H_2 Comparison of relevant properties	Expected ratio τ_{D2}/τ_{H2}	Times (in units of 10^{-12} sec) τ_{H2}	τ_{D2} Expected	τ_{D2} Measured (this experiment)
$\tau_\alpha(v_0)$	$v_0=0.05$ to $v_0=\alpha c=c/137$	Ionization and excitation.	Frequency of collisions.	Atomic density D_2 $=1.1\times$atomic density H_2.	0.9	3.3 ± 0.6[a]	3.0 ± 0.6	3.7 ± 0.7
$\tau_n(\alpha)$	$v=\alpha c$ to $v\approx10^{-4}c$	Ionization of molecules and elastic Coulomb collisions.	Frequency of collisions. Energy loss per collision.	Atomic density $D_2=1.1$ $\times$atomic density H_2. Nuclear mass D_2 $=2\times$nuclear mass H_2.	0.9 2.0 } 1.3	0.74	1.0	
	$(\pi^-H_2)^+$ to $\pi p(n\approx30)$ or $(\pi^-D_2)^+$ to $\pi d(n\approx30)$	Dissociation of $(\pi^-H_2)^+$ or $(\pi^-D_2)^+$	...	...		$\leqslant0.1$	$\leqslant0.1$	
	$n\approx30$ to $n\approx15$	Collisions with molecules.	Frequency of collisions. Energy loss per collision.	Atomic density D_2 $=1.1\times$atomic density H_2. Ratio of dissociation energies: $D_2/H_2=(4.476\text{ eV}/4.553\text{ eV})$	0.9 0.98 } 0.9	0.2	0.2	
	$n\approx15$ to $n\approx4$	Collisional Auger transitions.	Frequency of collisions. Reduced mass.	Atomic density D_2 $=1.1\times$atomic density H_2. $\left[\frac{\mu_p}{\mu_d}\right]^2=\left[\frac{m_\pi+m_d}{2(m_\pi+m_p)}\right]^2$	0.9 1.14 } 1.03	1.25	1.3	
						$\tau_n(\alpha)=2.2$	$\tau_n(\alpha)=2.5$	
$\tau_c(n\lesssim4)$	$n=4$ or $n=3$ to nuclear capture.	Stark mixing of angular momentum sublevels (rate $\propto n^{-2}$).	Frequency of collisions.	Atomic density D_2 $=1.1\times$atomic density H_2. Velocity of mesonic atom: $v_{\pi d}\approx v_{\pi p}/\sqrt{2}$. $\lambda_{ST(D_2)}(n)/\lambda_{ST(H_2)}(n)=0.77$		1.5[d]	1.7 (for $R=0.80$)	
		Nuclear capture.	Depletion of s states. s-state capture rate.	R from low-energy sections[b] $=0.80$. R from direct calculation[c] $=1.32$	$R_{eff}=1.20$ $R_{eff}=1.06$		1.5 (for $R=1.32$)	
$\tau_c(\alpha)$ $=\tau_n(\alpha)+\tau_c(n)$	$v_0=\alpha c$ to nuclear capture.	...	...	$\tau_{c(D)}(\alpha)$ (for $R=0.80$) $=1.1\tau_{n(H)}(\alpha)+1.20\tau_{c(H)}(n)$ $\tau_{c(D)}(\alpha)$ (for $R=1.32$) $=1.1\tau_{n(H)}(\alpha)+1.05\tau_{c(H)}(N)$	1.14 1.08	2.3 ± 0.6[a]	2.6 (for $R=0.80$) 2.5 (for $R=1.32$)	2.1 ± 0.5

[a] See Ref. 2.
[b] See Eq. (12) of text.
[c] See Eq. (15) of text and Ref. 18.
[d] See Ref. 7.

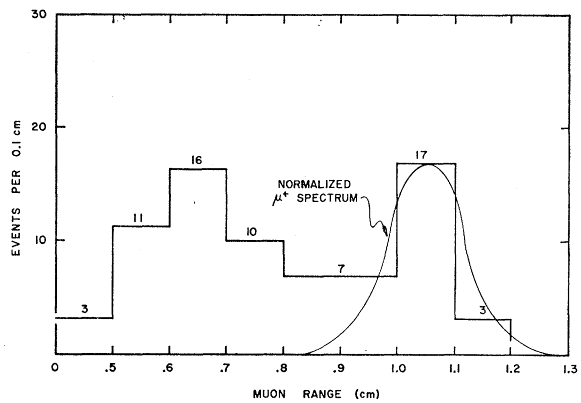

FIG. 2. Range spectrum of muons from $\pi^- - \mu^- - e^-$ events with laboratory angles $>110°$. The normalized muon spectrum from $\pi^+ - \mu^+ - e^+$ is superimposed.

ting points on a graph of range versus θ, as in Fig. 1, for a sample of positive pion decays, all of which may be assumed to occur at rest, and observing the fraction of points which fall in higher velocity intervals. Following this method, we find that the 21 negative pion decays with $v_0 \lesssim 0.006c$ shown in Fig. 1 should be increased to 24, the number with v_0 between $0.006c$ and $0.01c$ should be decreased from 2 to 0, and the number with v_0 between $0.01c$ and $0.02c$ should be decreased from 6 to 5.

The possibility of systematic errors due to (1) pion and muon scattering, (2) nuclear capture of the decay muon by deuterium or impurities, (3) nuclear capture of the pion by impurities, and (4) muon decay in flight have been considered and found to be negligible in all cases.

III. RESULTS

Our results are presented in Tables II and III. Table III gives the mean time for a negative pion to go from a velocity $v_0(=\beta_0 c)$ to nuclear capture in liquid deuterium of density 12.64×10^{-2} g/cm³. Table III also compares these mean times with those for liquid hydrogen in Ref. 2. The experimental error indicated for each interval in these tables combines (1) the uncertainty in the value of β for a given event

TABLE III. Comparison of moderation and absorption times (units of 10^{-12} sec).

Interval to nuclear capture	Liquid hydrogen[a]	Liquid deuterium[b]
$\beta_\pi = 0.006$ ($\approx\alpha$)	2.3 ± 0.6	$2.1\ \pm0.5$
$\beta_\pi = 0.01$	2.6 ± 0.6	$2.1\ \pm0.5$
$\beta_\pi = 0.02$	3.4 ± 0.6	$2.5\ \pm0.5$
$\beta_\pi = 0.03$	3.8 ± 0.6	3.16 ± 0.5
$\beta_\pi = 0.04$	4.6 ± 0.6	4.13 ± 0.63
$\beta_\pi = 0.05$	5.6 ± 0.8	5.76 ± 0.8

[a] As reported in Ref. 2.
[b] Liquid deuterium of density 12.64×10^{-2} g/cm³.

due to the uncertainty in the muon range, and (2) the statistical uncertainty in the number of pions found with β in a given interval.

The last row of Table II [the row labeled $\tau_c(\alpha)$] compares our results with values calculated using the figures for $R_{\rm eff}$ presented in Sec. I. Our figure for $\tau_{c(\rm D)}(\alpha)$, $2.1\pm0.5\times10^{-12}$ sec, is lower than the expected value, 2.6×10^{-12} sec, but the difference is within the range of experimental error. The measured ratio $[\tau_{c(\rm D)}(\alpha)/\tau_{c(\rm H)}(\alpha)]=0.90\pm0.30$ is also in agreement with the expected value 1.05.

We assume that the cascade process for kaons stopping in liquid hydrogen or liquid deuterium follows the general outline given in Sec. I for pions. The principal difference to be expected is that for kaons the nuclear capture rate $\Lambda_s n^{-3}$ should become comparable to the rate of Stark mixing $\Lambda_{\rm St}(n)$ at higher values of the principal quantum number ($n\approx20$ instead of $n\approx4$). The number of kaons captured will be approximately the same for many values of n, and for all these levels the effective capture rate will be controlled almost entirely by the rate of Stark mixing.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the assistance of our associates in the Bubble Chamber Group, E. Denton, P. Kloeppel, and S. Lucero. The pion exposure and event measurement were done at the Enrico Fermi Institute for Nuclear Studies of the University of Chicago. The analysis was done at Argonne National Laboratory.